Differential Decorrelator: A New Approach for Designing CDMA Linear Detector

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Abstract: Code Division Multiple Access is a widely used multiple access method in a lot of nowadays vital applications. The systems that are designed based on **CDMA** are suffering from a multiple access interference problem [1]. The decorrelator **CDMA** detector is a linear detector that can cancel the **MAI** but with some channel noise enhancement. The complexity of the decorrelator is a linear function in the number of system's users [2]. In this research, a new detector is proposed that can cancel the **MAI** in the received **CDMA** signal with a complexity that is independent on the number of system's users. The new detector does not need to know the users' signature codes. Also it consists of two matched filter only. No correlation matrix is required. This simple structure reduces the complexity of the proposed **CDMA** detector if it is compared with the conventional decorrelator detector. So, the system capacity can be increased where from the system implementation point of view, the detector structure is simple and it does not limit the number of users in the system.

The new detector is based on some mathematical operations on the output signals from two different matched filters. The detector idea is based on the symmetry property of the signatures' codes correlation matrix however it does not need to know this matrix. The algorithm is valid as long as the correlation matrix is symmetry. So it can work with synchronous and asynchronous system models.

Keywords: Code division multiple access – multiple access interference – matched filter – decorrelator detector – minimum mean square error detector – signature codes correlation matrix.

1 Introduction

Linear CDMA detectors are widely used in CDMA receivers' design because these detectors have a complexity that is linear with the number of system's users [2]. The most known linear CDMA detectors are Matched filter, Decorrelator, and MMSE adaptive filter. It is well known that CDMA system is interference limited system where the multiple access interference signals from the system's users that affect the desired user signal, are the most influential factor on the performance of this desired user signal [3]. Matched filter detector is the simplest CDMA detector. It is the optimum receiver of a known signal in AWGN environment [1]. But in CDMA system, the matched filter is not optimum because of the presence of system's MAI signals [4]. So the matched filter can be considered as the worst linear CDMA detector in the presence of high system interference signals' power. On the other hand, the decorrelator detector is the linear CDMA detector that completely cancels the MAI signals at the output of the detector [2]. However the decorrelator detector enhances the Gaussian noise power at the detector output. Also, the decorrelator structure is quite complex where it should know all the signatures' codes of all system users to form the decorrelation matrix which represents the inverse of the cross-correlation matrix among the system users' signatures' codes. Another source of complexity is the requirement of a bank of K-matched filters where K represents the number of system's users. These matched filters are matched to the signature codes of the K users in the CDMA system. The vector of K-users' energies that produces from the matched filters bank, will be multiplied by the decorrelation matrix [5]. Consequently the complexity of decorrelator detector is greater than the complexity of the matched filter detector. The MMSE detector is an adaptive algorithm detector that compromises between the matched filter detector and the decorrelator detector [6]. The MMSE detector minimizes the MAI signals' powers and the noise power jointly at the output of the detector. The MMSE detector needs to know the desired user signature code only. So MMSE detector's structure is simpler than the structure of decorrelator detector. But the MMSE detector is still complex with respect to matched filter detector. MMSE detector uses training sequence in the initiation of the communication link to adjust the MMSE adaptive filter taps then, the adaptive algorithm is working in decision directed mode to minimize the MMSE between the income signal and the detector output. The disadvantages of MMSE detector is its sensitivity to any abrupt change on the channel.

In this paper, a new linear CDMA detector is proposed to come up to the performance of the decorrelator detector but with simpler structure as matched filter detector. The proposed detector is based on a mathematical observation related to the symmetry property of the cross-correlation matrix among the CDMA system users' signature codes [7]. This new detector may help in increasing CDMA system capacity by allowing more number of system's users to share the same CDMA system's resources.

The remainder of this paper is organized as follows. In section (2), the system mathematical model is represented. This model helped in understanding the system behavior and the problems that are faced. The main idea of the new detector is shown in Section (3). Also, the new detector structure is represented in this section. The probability of error of the new detector's output is shown in section (4). This probability of error is compared with the probability of error in the cases of matched filter and decorrelator detectors. In section (5), the simulation results of the proposed detector are shown. The simulations results contain comparisons between the proposed detector and the other standard linear CDMA detectors. These comparisons use different two criterions to have fair judgment on the new proposed detector performance. Finally the conclusions and future works are contained in section (6).

2 Signal Model

Multiuser **CDMA** detectors commonly have a front end whose objective is to obtain a discrete time process from the received continuous time waveform y(t).

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma\, n(t), \qquad t \in [0, T] \qquad (1)$$

The notation introduced in Eq. (1) is defined as followed.

- **T** is the inverse of the data rate.
- $\mathbf{s_k}(\boldsymbol{t})$ is the deterministic signature waveform assigned to the $\boldsymbol{k}^{\text{th}}$ user, normalized so as to have unit energy.

$$\|s_k\|^2 = \int_0^T s_k^2(t).dt = 1 \qquad (2)$$

The signature waveform are assumed to be zero outside the interval [0, T], and therefore, there is no intersymbol interference.

- $\mathbf{A_k}$ is the received gain of the linear time invariant channel for user $\boldsymbol{k}$. $\mathbf{A_k}^2$ is referred to as the energy of the $\boldsymbol{k}^{th}$ user.
- $b_k \in [-1,1]$ is the bit transmitted by the $\boldsymbol{k}^{th}$ user.
- $\boldsymbol{n(t)}$ is white Gaussian noise with unit power spectral density. It models thermal noise plus other noise source unrelated to the transmitted signal. According to Eq. (1) the noise power in a frequency band B is $2\sigma^2$B.

Continuous to discrete time conversion can be realized by conventional sampling, or more generally, by correlation of y(t) with deterministic signals [2]. Two types of deterministic signals are of principal interest; the signature waveform and orthonormal signals [1].

One way of converting the received waveform into a discrete time process is to pass it through a bank of matched filters as shown in Fig.1. Each filter is matched to the signature waveform of a different user. In the synchronous case, the output of the bank of matched filter is shown in Eq. (3).

$$y_1 = \int_0^T y(t).s_1(t).dt\ ,\ \ y_2 = \int_0^T y(t).s_2(t).dt\ ,.........,\ \ y_K = \int_0^T y(t).s_K(t).dt \qquad (3)$$

where y(t) is represented by Eq. (1). The output of the $\boldsymbol{k}^{th}$ matched filter can be expressed as in Eq. (4).

$$y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k \qquad (4)$$

where:

$$\rho_{jk} = < s_j(t), s_k(t) > = \int_0^T s_j(t).s_k(t).dt \qquad (5)$$

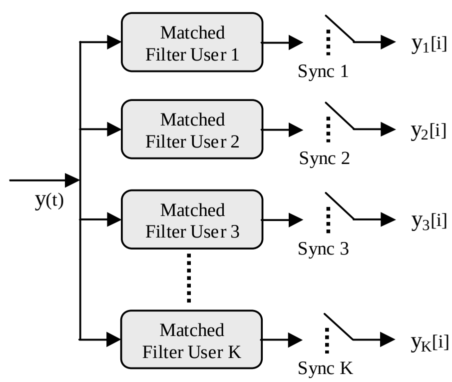

Figure 1 Discrete time K-dimensional vector of matched filter output.

$$n_k = \sigma \int_0^T n(t).s_k(t).dt \qquad (6)$$

It is noted that by Cauchy- Schwarz inequality and Eq. (2), the absolute value of the correlation coefficient is given in Eq. (7).

$$\left|\rho_{jk}\right| = \left| < s_j(t), s_k(t) > \right| \leq \left\|s_j\right\| \left\|s_k\right\| \qquad (7)$$

n_k is a Gaussian random variable with zero mean and variance equals to σ^2. It is convenient to express Eq. (4) in a vector form:

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \qquad (8)$$

where $\mathbf{R}=\{\rho_{jk}\}=\{<s_j(t),s_k(t)>\}$ is the normalized cross-correlation matrix. $\mathbf{y} = [\mathbf{y_1}, \mathbf{y_2}, \ldots, \mathbf{y_K}]^\mathbf{T}$, $\mathbf{b} = [\mathbf{b_1}, \mathbf{b_2}, \ldots, \mathbf{b_K}]^\mathbf{T}$, $\mathbf{A} = \mathbf{diag}[\mathbf{A_1}, \mathbf{A_2}, \ldots, \mathbf{A_K}]^\mathbf{T}$, and $\mathbf{n}$ is a zero mean Gaussian random vector with covariance matrix equals to:

$$E[\mathbf{nn}^T] = \sigma^2 \mathbf{R} \qquad (9)$$

No information related to the demodulation is lost by using the bank of matched filters; in other words, $y(t)$ can be replaced by **"y"** which is the ***sufficient statistic*** for the detection of users' data without loss of optimality [7].
To analyze any detector whose front end consists of a bank of matched filters, the original channel model can be replaced by the linear Gaussian K-dimensional model in Eq. (8). Recall that in the synchronous model it is sufficient to restrict attention to a one shot model; thus, the dependence of **y**, **b**, and **n** on the symbol index has been omitted.

3 Proposed CDMA Linear Detector

From the above discussion on the linear multiuser **CDMA** detectors, it is concluded that the complexity of the detector is increased if the capability of the detector to cancel the **MAI** is increased. The simplest **CDMA** detector is the matched filter. But the matched filter can not cancel the multiple access interference signals as shown in Eq. (10).

$$\underbrace{y_k}_{Detector\ output} = \underbrace{A_k b_k}_{Desired\ signal} + \underbrace{\sum_{j \neq k} A_j b_j \rho_{jk}}_{MAI} + \underbrace{n_k}_{Noise} \qquad (10)$$

The decorrelator detector can cancel the **MAI** signals completely but the structure of this detector needs to know the entire signature codes of the system's users. The decorrelator detector has a matched filter for each user signature. To cancel the **MAI** signal from the desired user signal, the decorrelator detector multiplies the output vector from the matched filters bank by the inverse of the cross-correlation matrix of the system's users. The decorrelator detector can cancel all **MAI** signals but it enhances the channel noise. Eq. (11) shows the decorrelator detector operation [8].

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{R}^{-1}\mathbf{RAb} + \mathbf{R}^{-1}\mathbf{n} = \mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n} \qquad (11)$$

Here the complexity of the decorrelator is linear with the number of **CDMA** system's users. But is it possible to have a detector that can cancel all the **MAI** signals with a complexity that is not depending on the number of users in the **CDMA** system?

The differential decorrelator is the detector that may answer the previous question. The receiver structure idea is base on the symmetry property of the signatures' codes correlation matrix [7]. The following equation represents the symmetry property of the correlation matrix.

$$\rho_{ij} = \rho_{ik} \quad For\ all\ \ 0 < i \,\&\, j \,\&\, k < K \qquad (12)$$

where $\boldsymbol{\rho_{ij}}$ is the correlation coefficient between user $\boldsymbol{i}$ and user $\boldsymbol{j}$ signatures' codes and it represents the element at row $\boldsymbol{i}$ and column $\boldsymbol{j}$ in the signatures' codes correlation matrix **R**. The signatures' codes correlation matrix **R** can be represented as:

$$\mathbf{R} = \mathbf{S}^H.\mathbf{S} = \begin{bmatrix} s_1^H \\ s_2^H \\ \vdots \\ s_K^H \end{bmatrix} \begin{bmatrix} s_1 & s_2 & \cdots & s_K \end{bmatrix} \qquad (13)$$

The operation of the differential decorrelator detector is based on using two matched filters and the symmetry property of the signatures' codes correlation matrix to eliminate the **MAI** signals. The first used filter is matched to the desired user signature code. It correlates the received signal with the signature code of the desired user. The second filter is the reference matched filter. This filter is matched to a reference signature code that is not used by any user in the system. The function of this second matched filter is to produce a **MAI** signal at its output whom is the same as the **MAI** produced at the desired user matched filter output. This matched filter is common in all receivers that use the working system. Eqs. (14-15) represent the output of the desired user (user ***k***) matched filter and the reference matched filter respectively.

$$y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k \qquad (14)$$

$$y_r = A_k b_k \rho_{rk} + \sum_{j \neq k} A_j b_j \rho_{jr} + n_r \qquad (15)$$

where:

$$\rho_{jk} = <s_j(t), s_k(t)> = \int_0^T s_j(t).s_k(t).dt \;\; ; \;\; n_k = \sigma \int_0^T n(t).s_k(t).dt$$

$$\rho_{jr} = <s_j(t), s_r(t)> = \int_0^T s_j(t).s_r(t).dt \;\; ; \;\; n_r = \sigma \int_0^T n(t).s_r(t).dt$$

and $\boldsymbol{s_k(t)}$ is the desired user signature code and $\boldsymbol{s_r(t)}$ is the reference signature code.

From the symmetry property of the correlation signatures' codes matrix **R**, it was found that:

$$\rho_{jk} = \rho_{jr} \quad For \;\; all \;\; 0 < j \,\&\, k < K \qquad (16)$$

So, by subtracting Eq. (15) from Eq. (14), it was found that:

$$y_k - y_r = A_k b_k (1 - \rho_{rk}) + n_k + n_r \qquad (17)$$

Eq. (17) represents the output of the differential decorrelator detector which is the decision statistic of the detector.

$$DS = A_k b_k (1 - \rho_{rk}) + n_k + n_r \qquad (18)$$

Now it is cleared that the proposed differential decorrelator detector has canceled all the **MAI** signals but on the cost of duplicating the back ground channel Gaussian noise.

The detector output which represents the estimate of the desired user data will be the sign of the decision statistic as in Eq. (19).

$$\hat{b}_k = \text{sgn}(DS) = \text{sgn}(A_k b_k (1 - \rho_{rk}) + n_k + n_r) \qquad (19)$$

Fig.2 shows the proposed differential decorrelator detector structure.

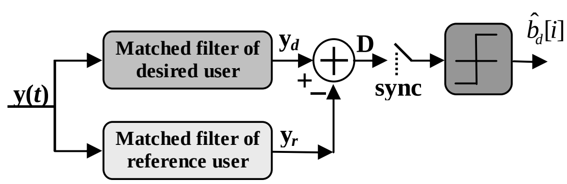

Figure 2 Proposed two signatures decorrelator detector structure.

The advantages of the differential decorrelator detector are the simple structures where the detector consists of two matched filters only instead of ***K*** matched filters as in the conventional decorrelator detector, the detector does not need to know the number of system's users nor their signatures' codes, and there is no need to neither calculate the inversion of the signatures' codes correlation matrix nor facing the problem of matrix singularity.

The disadvantage of differential decorrelator detector is the noise enhancing; the noise power is increased by 3dB due to the duplication of noise component in the decision statistic.

As it was shown, the differential decorrelator detector is based on the idea of symmetry property of the correlation matrix of the **CDMA** signatures' codes. But if this condition is not satisfied for certain **CDMA** signatures' codes family, what will be the solution in this case?

The previous problem's solution is not difficult. By using matrix algebra, the calculation of the reference signature code will be simple. For any working system, the correlation vector is calculated first between the desired user signature code and the other system signatures' codes as shown in Eq. (20).

$$\begin{bmatrix} s_1^H \\ s_2^H \\ \vdots \\ s_d^H \\ \vdots \\ s_K^H \end{bmatrix} .s_d = \begin{bmatrix} \rho_{d1} \\ \rho_{d2} \\ \vdots \\ 1 \\ \vdots \\ \rho_{dK} \end{bmatrix} \Rightarrow \Theta.s_d = \Delta \qquad (20)$$

where Θ is the systems' signature codes matrix, $\boldsymbol{s_d}$ is the desired user signature code vector, and Δ is the correlation between the desired user code and the other users' codes vector. The correlation vector Δ is used to calculate the reference signature code $\boldsymbol{s_r}$ after modifying the element of index **d** to be equaled to $\boldsymbol{\varepsilon}$ (small number) that represents the correlation between the desired user signature code and the reference signature code. Eq. (21) shows how the reference signature code can be calculated.

$$s_r = \Theta^{-1}.\Delta^{\backslash} \qquad where \quad \Delta^{\backslash} = \begin{bmatrix} \rho_{d1} \\ \rho_{d2} \\ \vdots \\ \varepsilon \\ \vdots \\ \rho_{dK} \end{bmatrix} \qquad (21)$$

In this case it is not necessary to have a common reference signature code for all systems receivers. On the other hand, each receiver may have its own reference signature code according to its desired user signature code as shown in Eqs. (20-21).

4 Performance of The proposed Detector in Linear AWGN Channel

The probability of error calculation is often depending on the decision variable which represents the output of the detector before the decision rule. This decision statistic represents a random variable called ***sufficient statistic***. For the desired user $\boldsymbol{k}$ that is a member in **CDMA** system, the probability of error in matched filter detector case is representing by Eq. (22) [1].

$$P_k^{mf}(\sigma) = \frac{1}{2} erfc\left(\frac{A_k}{\sqrt{2(\sigma^2 + \sum_{j \neq k} A_j^2 \rho_{jk}^2)}} \right) \qquad (22)$$

For the case of decorrelator detector, the desired user probability of error is representing by Eq. (23) [2].

$$P_k^d(\sigma) = \frac{1}{2} erfc\left(\frac{A_k}{\sigma\sqrt{2\mathbf{R}_{\mathbf{kk}}^{+}}} \right) = \frac{1}{2} erfc\left(\frac{A_k}{\sigma} \sqrt{\frac{\mathbf{1} - \mathbf{a}_{\mathbf{k}}^{\mathbf{T}} \mathbf{R}_{\mathbf{k}}^{-1} \mathbf{a}_{\mathbf{k}}}{2}} \right) \qquad (23)$$

where $\mathbf{a_k}$ is the $\boldsymbol{k}^{\text{th}}$ column of **R** without the diagonal element, and $\mathbf{R_k}$ is the (K-1)×(K-1) matrix that results by removing the $\boldsymbol{k}^{\text{th}}$ row and column from **R**. To calculate Eq. (23), the crosscorrelation matrix is assumed to be nonsingular.

The probability of error calculation in the case of the proposed differential decorrelator is very easy. By referring to Eq. (18), the proposed detector sufficient statistic can be written as in Eq. (24)

$$DS = A_k b_k (1 - \rho_{rk}) + n_{kr} \qquad (24)$$

where $\boldsymbol{n_{kr}}$ is a zero mean Gaussian noise with $2\sigma^2$ variance. So from Eq. (24), the proposed differential decorrelator probability of error can be represented as in Eq. (25).

$$P_k^{pd}(\sigma) = \frac{1}{2} erfc\left(\frac{A_k(1-\rho_{rk})}{2\sigma}\right) \qquad (25)$$

Fig.3 shows the plot of the probability of error verses signal to noise ratio in the desired user data at signal to interference ratio of -40 dB using Eqs. (22, 23, and 25) for matched filter detector, conventional decorrelator detector and proposed differential decorrelator detector respectively.

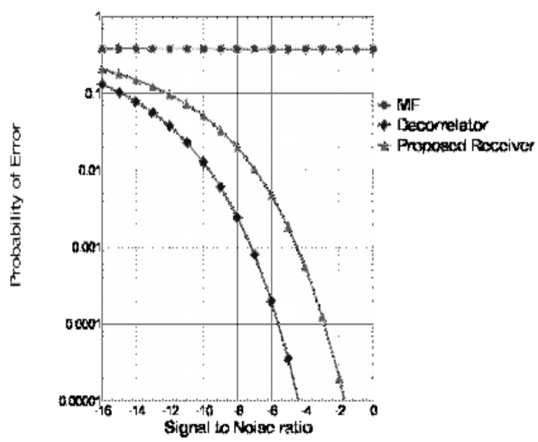

Figure 3 Probability of error for certain user in CDMA system using ML signature codes in linear time invariant channel and SIR=-40dB.

5 Performance of The proposed Detector in Non-linear Fading Channel

5.1 Nakagami flat fading channel

In Nakagami flat fading model, the desired user (user $\boldsymbol{k}$) has a complex Gaussian time varying channel gain $\boldsymbol{A_k}$. The amplitude of this gain represents Nakagami random variable and the phase of this gain represents uniform random variable. In any detector, the decision rule will be applied on real variable. So, the detector will calculate the real value of Eq.(24) before the **sign()** decision rule. Also, the detector estimates the time varying complex gain to be able to compensate the effect of channel fading. So the new decision statistic is formed as the output of the dot product between the detector output and the estimated channel gain $\hat{A}_k$ as shown in Eq.(26).

$$\begin{aligned} DS^m{}_k &= \hat{A}_k.(A_k b_k (1-\rho_{rk}) + n_{kr}) \qquad (26) \\ &= \left|\hat{A}_k\right|\left|A_k\right|\cos(\theta_e) b_k (1-\rho_{rk}) + \left|\hat{A}_k\right|\left|n_{kr}\right|\cos(\theta_{nk}) \end{aligned}$$

where $\boldsymbol{\theta_e}$ is the estimation phase error of user $\boldsymbol{k}$ and $\boldsymbol{\theta_{nk}}$ is the angle between the estimated complex channel gain vector and the complex Gaussian noise vector. The probability of error in the data which is estimated after applying the sign function on the modified decision statistic in Eq.(26) is represented by Eqs.(27-29).

$$P_k^{pd} = p\left[DS^{\backslash} > 0 \mid b_1 = -1\right]p\left[b_1 = -1\right] + p\left[DS^{\backslash} < 0 \mid b_k = +1\right]p\left[b_1 = +1\right] \quad (27)$$

$$P_k^{pd} = \frac{1}{2}p\left[-\left|\hat{A}_k\right|\left|A_k\right|\cos(\theta_e)b_k(1-\rho_{rk}) + \left|\hat{A}_k\right|\left|n_{kr}\right|\cos(\theta_{nk}) > 0 \mid b_1 = -1\right]$$

$$+\frac{1}{2}p\left[+\left|\hat{A}_k\right|\left|A_k\right|\cos(\theta_e)b_k(1-\rho_{rk}) + \left|\hat{A}_k\right|\left|n_{kr}\right|\cos(\theta_{nk}) < 0 \mid b_k = +1\right] \quad (28)$$

$$P_k^{pd} = \frac{1}{2}erfc\left(\frac{\left|A_k\right|(1-\rho_{rk})\cos(\theta_e)}{2\sigma\cos(\theta_{nk})}\right) \quad (29)$$

It should be noted that the term (**cos(*θ*nk))** is a random variable where $\boldsymbol{\theta_{nk}}$ is a uniform random variable that represents the angle difference between the noise vector and the estimated channel gain vector. In the worst case, this angle is equaled to zero. Also, the term (**cos($\boldsymbol{\theta_e}$)**) may be considered as a constant where good channel estimation algorithms can keep this factor constant and very small. So Eq.(29), for the worst case, can be written as:

$$P_k^{pd} = \frac{1}{2}erfc\left(\frac{\left|A_k\right|(1-\rho_{rk})\cos(\theta_e)}{2\sigma}\right) \quad (30)$$

Eq.(30) represents the probability of error of the proposed detector in Nakagami flat fading channel. But this probability of error is a random variable because $\left|A_k\right|$ is Nakagami random variable. So by assuming that($x = \left|A_k\right|$), the average of Eq.(30) is calculated as:

$$E[P_k^{pd}] = \int_{-\infty}^{\infty} P_k(x).f_x(x).dx \quad (31)$$

$$f_x(x) = \frac{2}{\Gamma(m)}.\left(\frac{m}{\Omega}\right)^m.x^{2m-1}e^{\left(\frac{-mx^2}{\Omega}\right)} \quad (32)$$

$$\Omega = E[x^2]$$

where $f_x(x)$ is the probability density function of Nakagami *m*-distribution random variable. From the properties of erfc(.) in [2], the following integration solution is represented.

$$\frac{1}{2}\int_0^{\infty} z^{2n-1}.e^{\left(\frac{-z^2}{2}\right)}.erfc\left(\frac{z}{\sqrt{2}\sigma}\right).dz =$$

$$= \frac{(n-1)!}{2}(1-(\sigma^2+1)^{-1/2})^n.\sum_{k=0}^{n-1}2^{-k}\binom{n-1+k}{k}(1+(\sigma^2+1)^{-1/2})^k \quad (33)$$

So, by using Eq.(30) and Eq.(33), the solution of Eq.(31) is represented as:

$$E[P_k^{pd}] = \frac{2^{-m}(m-1)!}{\Gamma(m)}(1-(v^2+1)^{-1/2})^m.\sum_{k=0}^{m-1}2^{-k}\binom{m-1+k}{k}(1+(v^2+1)^{-1/2})^k$$

$$v = \frac{\sqrt{2}\sigma}{(1-\rho_{rk})\cos(\theta_e)}\sqrt{\frac{2m}{\Omega}} \quad (34)$$

Eq.(34) represents the average probability of error of the two signature codes decorrelator (proposed detector) in flat fading Nakagami channel.

The same procedure is used to calculate the average probability of error for the decorrelator detector and binary signaling system (BPSK) in flat fading channel when channel phase estimation error is present. Eqs.(35-36) shows the average probability of error of decorrelator detector and binary signaling system respectively.

$$E[P_k^d] = \frac{2^{-m}(m-1)!}{\Gamma(m)}(1-(v^2+1)^{-1/2})^m.\sum_{k=0}^{m-1} 2^{-k}\binom{m-1+k}{k}(1+(v^2+1)^{-1/2})^k$$

$$v = \frac{\sigma}{\sqrt{\mathbf{1}-\mathbf{a}_\mathbf{k}^\mathbf{T}\mathbf{R}_\mathbf{k}^{-1}\mathbf{a}_\mathbf{k}}\ \cos(\theta_e)}\sqrt{\frac{2m}{\Omega}} \qquad (35)$$

$$E[P_k^{BPSK}] = \frac{2^{-m}(m-1)!}{\Gamma(m)}(1-(v^2+1)^{-1/2})^m.\sum_{k=0}^{m-1} 2^{-k}\binom{m-1+k}{k}(1+(v^2+1)^{-1/2})^k$$

$$v = \frac{\sigma}{\cos(\theta_e)}\sqrt{\frac{2m}{\Omega}} \qquad (36)$$

5.2 Nakagami multipath fading model

In the case of multipath channel model, the DS-CDMA received signal is represented as in Eq.(37).

$$r(t) = \sum_{k=1}^{K}\sum_{i=1}^{L} b_k A_{ki} s_k(t) + \sigma n(t) \qquad (37)$$

To detect the desired user signal, L-fingers rack receiver is used with MRC (Maximum Ratio Combiner). In each finger the proposed detector is used to eliminate the multiple access interference. After using the proposed detector, MRC is used to compensate the multipath fading effect. The output of the MRC is represented in Eq.(42).

$$DS_k = \sum_{i=1}^{L}\left|A_{ki}\right|^2(1-\rho_{kr})\cos(\theta_{ie})b_k + \sigma\sum_{i=1}^{L}\left|A_{ki}\right|(n_{ki}\cos(\theta_{nk}(i)) - n_{ri}\cos(\theta_{rk}(i))) \quad (38)$$

$$n_{ki} = e^{-j\theta_i}\int_0^T n(t).s_k^*(t).dt \qquad , \qquad n_{ri} = e^{-j\theta_i}\int_0^T n(t).s_r^*(t).dt$$

By following the same assumptions of θ_{ie}, $\theta_{nk}(i)$, and $\theta_{rk}(i)$ as in flat fading case, the mean and the variance values of the decision statistic in equation (38) can be represented as in Eqs.(39-40).

$$E[DS_k] = \sum_{i=1}^{L}\left|A_{ki}\right|^2\cos(\theta_{ie})(1-\rho_{kr}) \qquad (39)$$

$$E[DS_k^2 - E[DS_k]^2] = 2\sigma^2\sum_{i=1}^{L}\left|A_{ki}\right|^2 \qquad (40)$$

So, the probability of error can be represented as:

$$P_k^{pd} = \frac{1}{2}erfc\left(\frac{(1-\rho_{kr}).\sqrt{\sum_{i=1}^{L}\left|A_{ki}\right|^2\cos^2(\theta_{ie})}}{2\sigma}\right) \qquad (41)$$

The random variable $|A_{ki}|$ is assumed to be a m-distribution Nakagami random variable. The probability density function of $|A_{ki}|$ is given in Eq.(32). In another way, $|A_{ki}|$ can be considered as a Rayleigh random variable with 2m degree of freedom. So $|A_{ki}|^2$ is a chi-square random variable with 2m degree of freedom too. If R_i=|

$A_{ki}|^2\cos^2(\theta_{ik})$, the probability density function and characteristic function of R_i will be given as in Eq.(42) and Eq.(43) respectively.

$$f_{R_i}(R_i) = \frac{1}{\Gamma(m_i)}\left(\frac{m_i}{\Omega_i \cos^2(\theta_{ie})}\right)^{m_i} .R_i^{\,m_i-1}.e^{-\frac{m_i}{\Omega_i \cos^2(\theta_{ie})}R_i} \qquad (42)$$

$$\Phi_{R_i}(S) = \left(\frac{m_i/\Omega_i \cos^2(\theta_{ie})}{S + m_i/\Omega_i \cos^2(\theta_{ie})}\right)^{m_i} \qquad (43)$$

By assuming that the random variables R_i (i=1→L) are statistically independent. So, the random variable $R = \sum_{i=1}^{L} R_i$ has a characteristic function equaled to:

$$\Phi_R(S) = \prod_{i=1}^{L}\left(\frac{m_i/\Omega_i \cos^2(\theta_{ie})}{S + m_i/\Omega_i \cos^2(\theta_{ie})}\right)^{m_i} \qquad (44)$$

The probability density function of R is represented by:

$$f_R(R) = \frac{1}{\Gamma(m_t)}\left(\frac{m_t}{\Omega_t}\right)^{m_t} R^{m_t-1} e^{-\frac{m_t}{\Omega_t}R} \qquad (45)$$

$$m_t = \sum_{i=1}^{L} m_i \qquad and \qquad \Omega_t = \sum_{i=1}^{L} \Omega_i \cos^2(\theta_{ie})$$

From Eq.(41) and Eq.(44), the average probability of error for the proposed detector in Nakagami multipath channel in given by:

$$E[P_k^{pd}] = \int_{-\infty}^{\infty} P_k . f_R(R).dR = \frac{2^{-m_t}(m_t-1)!}{\Gamma(m_t)}\left(1-(q^2+1)^{-\frac{1}{2}}\right)^{m_t} \sum_{i=1}^{m_t-1} 2^{-i}\binom{m_t-1+i}{i}\left(1+(q^2+1)^{-\frac{1}{2}}\right)^{i}$$

$$q = \frac{\sqrt{2}\sigma}{(1-\rho_{kr})}\sqrt{\frac{2m_t}{\Omega_t}} \qquad (46)$$

For the decorrelator detector, the same procedure of average probability of error calculations is followed. When channel phase estimation error is exist, the formula of decorrelator average probability of error in Nakagami multipath fading channel is given in Eq.(47).

$$E[P_k^{d}] = \int_{-\infty}^{\infty} P_k . f_R(R).dR = \frac{2^{-m_t}(m_t-1)!}{\Gamma(m_t)}\left(1-(q^2+1)^{-\frac{1}{2}}\right)^{m_t} \sum_{i=1}^{m_t-1} 2^{-i}\binom{m_t-1+i}{i}\left(1+(q^2+1)^{-\frac{1}{2}}\right)^{i}$$

$$q = \frac{\sigma}{\sqrt{\mathbf{1}-\mathbf{a}_\mathbf{k}^\mathbf{T}\mathbf{R}_\mathbf{k}^{-1}\mathbf{a}_\mathbf{k}}}\sqrt{\frac{2m_t}{\Omega_t}} \qquad (47)$$

By the same way, the average probability of error for BPSK in multipath Nakagami fading channel is represented in Eq.(48) when channel phase estimation error is exist.

$$E[P_k^{BPSK}] = \int_{-\infty}^{\infty} P_k . f_R(R).dR =$$

$$= \frac{2^{-m_t}(m_t-1)!}{\Gamma(m_t)}\left(1-\left(\frac{2m_t\sigma^2}{\Omega_t}+1\right)^{-\frac{1}{2}}\right)^{m_t} \sum_{i=1}^{m_t-1} 2^{-i}\binom{m_t-1+i}{i}\left(1+\left(\frac{2m_t\sigma^2}{\Omega_t}+1\right)^{-\frac{1}{2}}\right)^{i} \qquad (48)$$

6 Simulation Results

In this section, the linear **CDMA** multiuser detectors' models that were represented in communication literature have been used here to compare the performance of these linear multiuser detectors simulation models in linear time invariant channel. The used **CDMA** multiuser detectors models are matched filter (MF) detector, decorrelator detector, minimum mean square error (MMSE) adaptive algorithms' detectors such as least mean square (LMS) algorithm and recursive least squares (RLS) algorithm [9], differential minimum mean square error (DMMSE) adaptive algorithm detector [10], Kalman adaptive filter detector [11, 12], and the proposed differential decorrelator detector.

The performance investigation is done using two different criterions. The average bit error rate criterion and the interference power measurement at the detectors' outputs. Two different signal to interference ratios are used at the input of each **CDMA** multiuser detector (-20 dB & -40dB). For each value of these input signal-to-interference ratios, the multiple access interference power has been measured at each detector output.

These two different criterions help in putting up a complete clear view on the performance of the linear **CDMA** multiuser detectors that have been used in a lot of **CDMA** networks. Also, they will help in the comparison with the new proposed one.

The simulations are done using maximal length signature codes. The simulations are done at two different SIR values at detectors' inputs. These SIR values are chosen to be smaller than –13 dB. From the CDG (**CDMA** Development Group) testing standards, the SIR value of –13dB is the common reference value of interference at **CDMA** detector input in any **CDMA** network [13]. The average received SNR value at different detector's inputs is varied from (– 30 dB) to (10 dB). The BER curves are plotted verses the average received signal to noise ratio for each signal to interference ratio.

Figs.4-5 show the bit error rate curves of the pre-mentioned standard multiuser **CDMA** detectors using a data packet of length **10^5** bits at different signal to noise ratios for linear time invariant channel. The users' signature codes are maximal length codes of period **31** chips. The input SIR values are **-20** dB and **-40** dB respectively. The average input SNR is varied between **-30** dB to **10** dB steps **2** dB. The used modulation scheme was **PSK** for coherent modulated system and **DPSK** for non-coherent modulated systems.

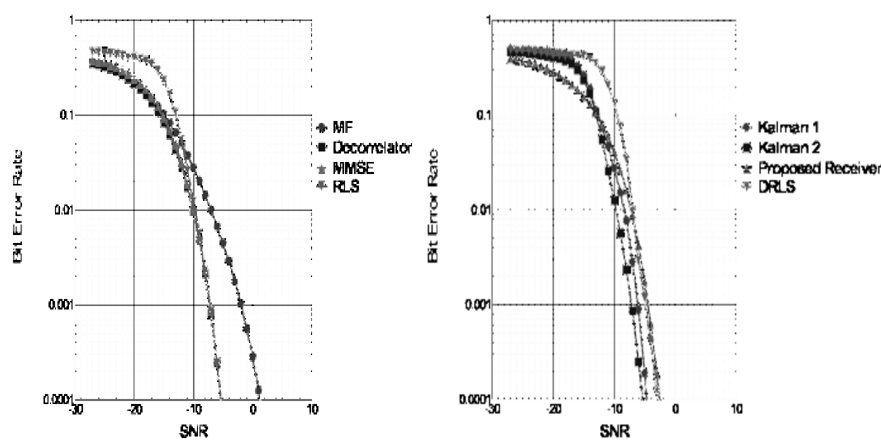

Figure 4 Bit error rate verses average signal to noise ratio for linear CDMA multi-user standard detectors in linear time invariant channel and SIR=-20dB.

From Fig.4, it was clear that at low signal to noise ratio, the BER of the matched filter, decorrelator and normalized MMSE detectors are better than the BER of RLS detector. On the other hand at high signal to noise ratio, the BER of the decorrelator, normalized MMSE, and RLS detectors have approximately the same BER and they are better than matched filter detector. Also, it was shown that the BER of the proposed detector is better than the BER of Kalman filters-1&2 and DRLS detectors at low signal to noise ratio. But in high signal to noise ratio, the BER of Kalman filter-2 detector is better than BER of Kalman filter-1 detector by approximately 1 dB and it is better than the BER of the proposed detector and DRLS detector by approximately 3 dB too.

In Fig.5, the effect of very low signal to interference ratio (-40dB) is appeared. The BER performance of matched filter detector is roughly constant and there is no enhancement in its performance with the increasing of the signal to noise ratio. However the other detectors give enhanced performance with the increasing of SNR. It is also shown that the BER performance of the conventional decorrelator and normalized MMSE detector are the same at low signal to noise ratio and they are better than the BER performance of RLS detector. On the other hand, at high signal to noise ratio, the BER performance of the conventional decorrelator and RLS detectors are approximately the same and they are better than the BER performance of the normalized MMSE detector with approximately 1 dB. Also, it was shown that the BER of the proposed detector is better than the BER of Kalman filters-1&2 and DRLS detectors at low signal to noise ratio. But in high signal to noise ratio, the BER of Kalman filter-2 detector is better than BER of Kalman filter-1 detector by approximately 1 dB and it is better than the BER of the proposed detector and DRLS detector by approximately 3 dB too.

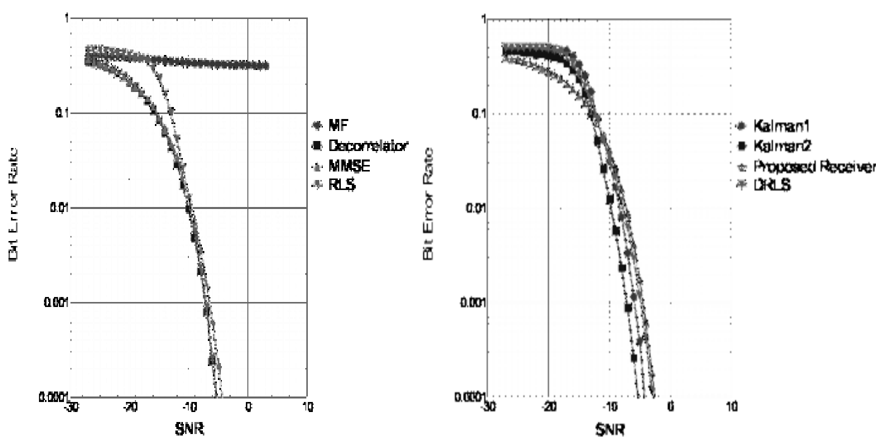

Figure 5 Bit error rate verses average signal to noise ratio for linear CDMA multi-user standard detectors in linear time invariant channel and SIR=-40dB.

Table 1 and 2 show the interference signals' power at the **CDMA** linear detectors' outputs. The calculations are done at the same simulation conditions as in the calculations of BER performance.

In table 1, it was shown that the interfering signal power value at decorrelator and proposed detector outputs are zero however all the other detectors have a significant interference signal power values at their outputs. The normalized MMSE, RLS, Kalman filter-2 and DRLS detectors have approximately the same interfering signal power value at their outputs. Kalman filter-1 detector output has an interfering signal power value greater than the interfering signal power value of the other detectors except the matched filter detector which has the highest interfering signal power value at its output.

Table.1 The interference signal power at the output of linear CDMA multi-user standard detectors in linear time invariant channel and SIR=-20dB.

Detector	Normalized interference power at detector output	Detector	Normalized interference power at detector output
MF	0.010470	Kalman filter 1	0.0116
Decorrelator	0	Kalman filter 2	0.0011
MMSE	0.0012	Proposed detector	0
RLS	0.0011	DRLS	0.0008

In table 2, it was shown that the interfering signal power value at decorrelator and proposed detector outputs are also zero however all the other detectors have a approximately the same interference signal power values at their outputs except the matched filter detector which has the highest interfering signal power value at its output.

Table.2 The interference signal power at the output of linear CDMA multi-user standard detectors in linear time invariant channel and SIR=-40dB.

Detector	Normalized interference power at detector output	Detector	Normalized interference power at detector output
MF	0.104230	Kalman filter 1	0.1007
Decorrelator	0	Kalman filter 2	0.1008
MMSE	0.1012	Proposed detector	0
RLS	0.1008	DRLS	0.005

For Nakagami fading channel, simulations are done on three different detectors; binary signaling (BPSK), Decorrelator detector, and the proposed detector. The used channel estimation bases on normalized LMS algorithm. Simulations are done at 5% phase error in channel phase estimation. Fig.6 shows the average probabilities of errors' comparisons in the cases of BPSK, the decorrelator detector, and the proposed detector for Nakagami flat fading channel of ***m***=1, 2, &4.

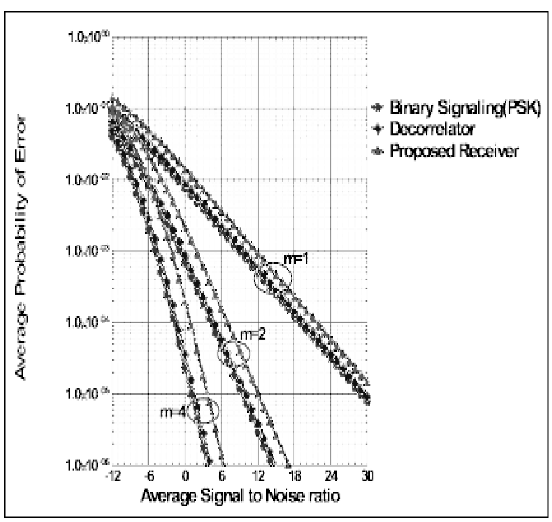

Figure 6 Average probability of error for certain user in CDMA system using ML signature codes in Nakagami flat fading channel with *m*=1, 2, &4 and SIR=-40dB.

Figs 7-9 show the average probability of error for the decorrelator detector, the proposed detector, and the BPSK receiver in Nakagami multipath channel with m=1, 2, &4 and L=1, 2, &4.

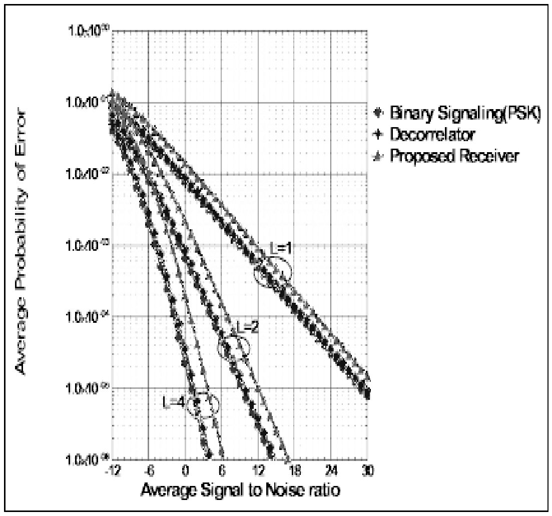

Figure 7 Average probability of error for certain user in CDMA system using ML signature codes in Nakagami multipath fading channel with *m*=1, L=1, 2, &4 and SIR=-40dB.

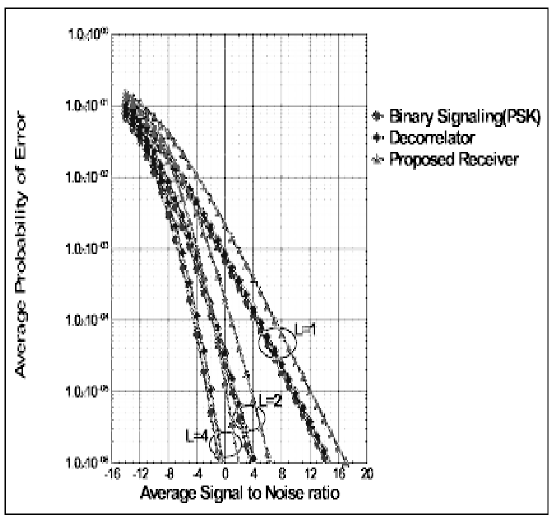

Figure 8 Average probability of error for certain user in CDMA system using ML signature codes in Nakagami multipath fading channel with *m*=2, L=1, 2, &4 and SIR=-40dB.

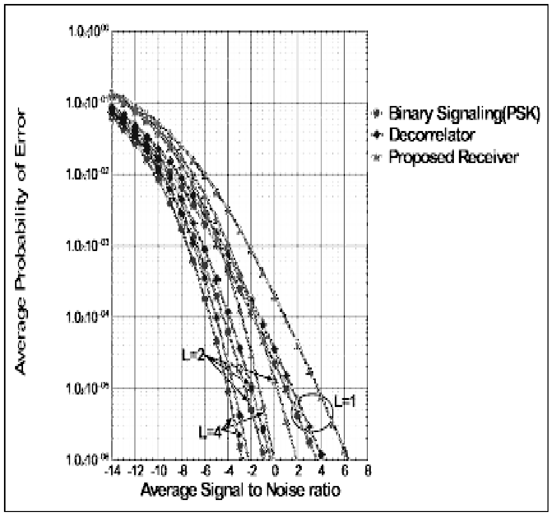

Figure 9 Average probability of error for certain user in CDMA system using ML signature codes in Nakagami multipath fading channel with m=4, L=1, 2, &4 and SIR=-40dB.

7 Conclusions and Future Work

The novel of this new **CDMA** detector is the capability of **MAI** cancellation with a simple structure that is independent on the number of system users. The new detector works with two matched filter only however in the decorrelator detector the number of used matched filters equals the number of system users. The new detector needs to know the desired user signature code with the reference code although the decorrelator detector needs to know all the signatures' codes of the users in the system. The proposed detector does not need to know the correlation matrix between the users' codes. It requires only having a symmetric correlation matrix. Even if the decorrelator detector requires knowing the correlation matrix of the users' signatures codes and it also wants to calculate its inverse. So, the new proposed **CDMA** detector has a simple structure as a matched filter but its **MAI** cancellation capability is similar to the decorrelator detector.

The new proposed detector has the disadvantage of channel noise enhancement as the decorrelator detector. But in the decorrelator detector, the noise enhancement is variable and it is a function of the norm of the correlation matrix inverse as shown in Eq. (27). However the proposed detector has a fixed amount of channel noise enhancement. This noise enhancement equals to the normal channel noise power.

This paper also represents a mathematical formula to the probability of error for the proposed detector in AWGN linear time invariant channel. Also, different performance comparisons have been made between the proposed detector and other standard **CDMA** detectors mathematically and through simulations. The comparisons guarantee the **MAI** cancellation capability of the proposed detector. Also they show that the performance of the proposed detector is as good as the performance of the other standard **CDMA** detectors in **MAI** cancellation other than the complexity of the proposed detector is the simplest among the other detectors that achieve the same performance.

The operation of CDMA systems in time varying channel has two main problems. They are MAI and the time varying channel estimation. The solution of one problem is affected by the existence of the other. A lot of solutions have been set to overcome these two problem but with complex structure. The differential detector that has been proposed in this paper can cancel the MAI with simple structure as the matched filter. The cancellation of MAI before the channel estimation is better because in this case the MAI will not affect the channel estimation process. By this new receiver structure, the channel estimation will not depend on the MAI and the channel's complex gains estimation will be more accurate.

The paper also represents a mathematical study of the proposed detector in Nakagami time varying channel. The performance of the proposed detector has been evaluated mathematically in flat fading model and multipath fading model. These equations are useful on the design of any CDMA system based on the proposed detector in Nakagami channel.

Channel phase estimation error has been included in the probability of error calculations. This is an important parameter where the performance of the system is affected significantly by it. It was shown that the channel phase estimation error is depending on the type of the used channel estimator.

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Biography

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